NUMBERS

I.Numbers: In Hindu-Arabic System, we use ten symbols, namely 0,1,2,3,4,5,6,7,8,and 9. we call them digits. A number is denoted by a group of digits, called numeral.

II. FACE VALUE OF A DIGIT IN A NUMBER The face value of a digit in a number is the value of the digits itself.

Example- The face value of 7 of the number 536749 is 7. The face value of 3 of the number 536749 is 3.

III. PLACE VALUE/ LOCAL VALUE OF A DIGIT IN A NUMBER.
Example- In the Number 265739, we have Place value of 5 is $(5 \times 1000) = 5000$ Place value of 3 is $(3 \times 10) = 30$ Place value of 2 is $(2 \times 100000) = 200000$

IV. TYPES OF NUMBERS.

(i) **Natural Numbers/Positive Integers:** Counting numbers are called natural numbers. Thus $\mathbf{N}=\{1, 2, 3, 4, 5, ...\}$

(ii) **Whole Numbers:** All natural numbers together with zero form the set of whole numbers. Notes:(i) 0 is a whole number which is not a natural number.
(ii) Every natural number is a whole number.

(iii) **Integers:**All positive integers, negative integers and Zero are called integers. $Z = \{... - 3, -2, -1, 0, 1, 2, 3, ...\}$ (a)negative Integers: $\{-1, -2, -3, ...\}$ is set of all negative integers. (b)Non- Negative integers=$\{0, 1, 2, 3, ...\}$
(c)Non- positive integers=$\{0, -1, -2, -3, ...\}$

(iv) **Rational Numbers:** A number is said to be rational, if it is of the form of $\frac{p}{q}, q \neq 0, HCF(p, q) = 1$, p, q are integers.
Examples: $\frac{1}{2}, \frac{-3}{16}$, etc.
NOTE: All integers are rational numbers.

(iv) **Irrational Numbers:** A number is said to be irrational, If it is non-terminating and non-repeated.
Examples: $\sqrt{2}, \sqrt{3}, \sqrt[3]{3}, \sqrt[3]{2}, (\sqrt{p}, p$ is prime), $\pi, e, etc.$

(iv) **Even Numbers:** A number divisible by 2 is called even number. colorblue Example: 0,2,4,6,8,10,... are even numbers.

(v) **Odd Numbers:** A number not divisible by 2 is called odd number. colorblue Example: 1,3,5,7,9,11,... are odd numbers.

(vi) **Prime Numbers:** A positive number greater than 1 having exactly two factors, i,e 1 and the number itself, is called a prime number.

examples of prime numbers upto 100:
2,3,5,7,,11,13,17,19,23,29,31,37,41,43,47,53,59, 61,67,71,73,79,83,89,97.

To Test Whether a given number 'p' is Prime:
Let 'p' be given number . Find a whole number 'k' such that $k > \sqrt{p}$. Take all prime numbers less than or equal to 'k'. If no one of these divides p exactly, we say 'p' is prime. Otherwise 'p' is not prime.

Example: Text whether 191 is prime or not?
Ans: clearly, $14^2 > 191$, here $k = 14$, Prime numbers upto 14 are 2,3,5,7,11,13. No one of these divides 191 exactly. there fore 191 is a prime number.

Example: Text whether 221 is prime or not?
Ans: clearly, $15^2 > 191$, here $k = 15$, Prime numbers upto 15 are 2,3,5,7,11,13. clearly 13 divides 221 exactly. therefore 221 is not a prime number.

(vii) **Co-Prime Numbers:** Two natural numbers are said to co-primes, if their HCF is 1,(i.e. Highest Common factor, HCF(a,b)=1)

Examples of Co-prime: $HCF(2,3) = 1 \Longrightarrow (2,3)$ are co-primes. $HCF(4,5) = 1 \Longrightarrow (4,5)$ are co-primes. $HCF(7,9) = 1 \Longrightarrow (7,9)$ are co-primes. $HCF(8,11) = 1 \Longrightarrow (8,11)$ are co-primes

(viii) **Composite Numbers:**
The Number greater than 1 which are not prime, are called composite number.
Example: 4,6,8,9,10,.....
NOTE:
(a) 1 is neither prime nor composite
(b) 2 is only even number, which is prime.

V. TEST OF DIVISIBILITY:

(a) Divisibility by '2':A number is divisible by '2', If its unit digit is any one of $0, 2, 4, 6, 8$.
Example: 64892 is divisible by '2', because its unit digit is 2. But 45361 is not divisible by 2, because its unit digit is not any one of $0, 2, 4, 6, 8$.

(b) Divisibility by '3': A number is divisible by 3 only when the sum of its digits is divisible by 3.
Example:.(a) Consider the number 587421.
sum of digits = 5 + 8 + 7 + 4 + 2 + 1 = 27, which is divisible by 3. So, 587421 is divisible by 3.
(b) Consider the number 689453.

Sum of digits = 6 + 8 + 9 + 4 + 5 + 3 = 35 is not divisible by 3. So, the number 689453 is not divisible by 3.

(c) Divisibility by '4': A number is divisible by 4, if the number formed by last two digits of that number is divisible by 4.
Example:(a) Consider the number 5249376. The number formed by last two digits is 76, which is divisible by 4. So, the number 5249376 is divisible by 4.
(b)Consider the number 638214. The number formed by last two digits is 14, which is not divisible by 4. So, the number 638214 is not divisible by 4.

(d) Divisibility by '5':A number is divisible by '5', If its unit digit is either 0 or 5 only. Example:(a)Consider the number 324575. Its unit digit is 5, So the number 324575 is divisible by 5.
(b) Consider the number 1796470. Its unit digit is 0, So the number 324575 is divisible by 5.

(d) Divisibility by '6':A number is divisible by 6 if it is divisible by both 2 and 3.
Example:(a)Consider the number 974562. Its unit digit is 2, So the number 974562 is divisible by 2.also sum of digits is 9 + 7 + 4 + 5 + 6 + 2 = 33, which is divisible by 3.so the number is divisible by 3.Thus the number is divisible by 6.
(b) Consider the number 975416. Its unit digit is 2, So the number 975416 is divisible by 2.Nut sum of digits=9 + 7 + 5 + 4 + 1 + 6 = 32, which is not divisible by 3. Thus the number is not divisible by 3.Hence the number is not divisible by 6.

(e) Divisibility by '8': A number is divisible by 8, if a number formed by last three digits of that number is divisible by 8.
Example:(a) Consider the number 5249320. The number formed by last three digits is 320, which is divisible by 8. So, the number 5249320 is divisible by 8.
(b)Consider the number 638214. The number formed by last three digits is 214, which is not divisible by 8. So, the number 638214 is not divisible by 8.

(f) Divisibility by '9': A number is divisible by 9 only when the sum of its digits is divisible by 9.
Example:.(a) Consider the number 587421.
sum of digits = 5 + 8 + 7 + 4 + 2 + 1 = 27, which is divisible by 9. So, 587421 is divisible by 9.
(b) Consider the number 689453.
Sum of digits = 6 + 8 + 9 + 4 + 5 + 3 = 35 is not divisible by 9. So, the number 689453 is not divisible by 9.

(g) Divisibility by '10':A number is divisible by '10', If its unit digit is 0 only.
Example:(a)Consider the number 324570. Its unit digit is 0, So the number 324570 is divisible by 10.
(b) Consider the number 1796475. Its unit digit is 5, So the number 1797475 is not divisible by 10.

(h) Divisibility by '11': A number is divisible by 11, if the difference of the sum of

its digits at odd place and the sum of its digits at even places is either 0 or a number divisible by 11
Example:(a)Consider the number 49235714. Its sum of odd place digits is (4 + 7 + 3 + 9 = 23) and sum of even place digits is (1 + 5 + 2 + 4 = 12). Then their difference is 23 − 12 = 11, which is divisible by 11. Thus the number 49235714 is divisible by 11.

(i) Divisibility by '7': **(Method)**
Unit digits of any number multiply by 5 and add in remaining part of the number. If the addition result is divisible by 7, then the given number is divisible by 7.
Example:(a)Consider the number 3983.
Unit digit= 3,
Now, 3 × 5 = 15.
remaining part of that number is = 398.
398 + 15 = 413,
Again its unit digit is 3.
3 × 5 = 15, remaining part is 41.
41 + 15 = 56, which is divisible by 7.
Thus the number 3983 is divisible by 7.

(j) Divisibility by '13': **(Method)**
Unit digits of any number multiply by 4 and add in remaining part of the number. If the addition result is divisible by 13, then the given number is divisible by 13.
Example:(a)Consider the number 14131.
Unit digit= 1,
Now, 1 × 4 = 4.
remaining part of that number is = 1413.
1413 + 4 = 1417,
Again its unit digit is 7.
7 × 4 = 28, remaining part is 141.
141 + 28 = 169, which is divisible by 13.
Thus the number 14131 is divisible by 13.

(k) Divisibility by '17': **(Method)**
Unit digits of any number multiply by 12 and add in remaining part of the number. If the addition result is divisible by 17, then the given number is divisible by 17.
Example:(a)Consider the number 18479.
Unit digit= 9,
Now, 9 × 12 = 108.
remaining part of that number is = 1847.
1847 + 108 = 1955,
Again its unit digit is 5.
5 × 12 = 60, remaining part is 195.
195 + 60 = 255, which is divisible by 17.
Thus the number 18479 is divisible by 17.

(l) Divisibility by '19': **(Method)**
Unit digits of any number multiply by 2 and add in remaining part of the number.

If the addition result is divisible by 19, then the given number is divisible by 19.
Example:(a)Consider the number 14326.
Unit digit= 6,
Now, $6 \times 2 = 12$.
remaining part of that number is = 1432.
$1432 + 12 = 1444$,
Again its unit digit is 4.
$4 \times 2 = 8$, remaining part is 144.
$144 + 8 = 152$, which is divisible by 19.
Thus the number 14326 is divisible by 19.

(m) Divisibility by '23': **(Method)**
Unit digits of any number multiply by 7 and add in remaining part of the number.
If the addition result is divisible by 23, then the given number is divisible by 23.
Example:(a)Consider the number 33971.
Unit digit= 1,
Now, $1 \times 7 = 7$.
remaining part of that number is = 3397.
$3397 + 7 = 3404$,
Again its unit digit is 4.
$4 \times 7 = 28$, remaining part is 340.
$340 + 28 = 368$,
again its unit digit is 8.
$8 \times 7 = 56$, remaining part is 36
$36 + 56 = 92$, which is divisible by 23.
Thus the number 33971 is divisible by 23.

(m) Divisibility by '29': **(Method)**
Unit digits of any number multiply by 3 and add in remaining part of the number.
If the addition result is divisible by 29, then the given number is divisible by 29.
Example:(a)Consider the number 25433.
Unit digit= 3,
Now, $3 \times 3 = 9$.
remaining part of that number is = 2543.
$2543 + 9 = 2552$,
Again its unit digit is 2.
$2 \times 3 = 6$, remaining part is 255.
$255 + 6 = 261$,
again its unit digit is 1.
$1 \times 3 = 3$, remaining part is 26
$26 + 3 = 29$, which is divisible by 29.
Thus the number 25433 is divisible by 29.

AN IMPORTANT NOTE: If a number N is divisible by two numbers $aandb$, where a and b are co-primes(HCF(a,b)=1), the N is divisible by 'ab' 1. If the number 517*324 is completely divisible by 3, then the smallest whole number in place of * will be....

(a) 0

(b) 1

(c) 2

(d) None of these

2. If the number 481*673 is completely divisible by 9, then the smallest whole number in place of * will be....

(a) 2

(b) 5

(c) 6

(d) 7

2. If the number 97215*6 is completely divisible by 11, then the smallest whole number in place of * will be....

(a) 3

(b) 2

(c) 1

(d) 5

3. If the number 91876*2 is completely divisible by 8, then the smallest whole number in place of * will be....

(a) 1

(b) 2

(c) 3

(d) 4

5. Which of the following number is divisible by 24?

(a) 35718

(b) 63810

(c) 537804

(d) 3125736

6. How many of the following numbers are divisible by 132.
264,396,462,792,968,2178,5184,6336

(a) 4

(b) 5

(c) 6

(d) 7

7. 476**0 is divisible by both 3 and 11. The nonzero digits in the hundred's and ten's places are respectively.

(a) 7 and 4

(b) 7 and 5

(c) 8 and 5

(d) None of these

9. If 5432*7 is divisible by 9, then the digit in place of * is

(a) 0

(b) 1

(c) 6

(d) 9

10. If 78*3945 is divisible by 11, then the digit in place of * is

(a) 0

(b) 1

(c) 3

(d) 5

11. Which one of the following number is completely divisible by 45

(a) 181560

(b) 331145

(c) 202860

(d) 2033550

12. Which one of the following number is completely divisible by 99

(a) 3572404

(b) 135792

(c) 913464

(d) 114345

13. If the number 42573* is exactly divisible by 72, then the minimum value in place of * is

(a) 4

(b) 5

(c) 6

(d) 7

14. If x and y are the two digits of the number 653xy such that this number is divisible by 80, then x+y=....

(a) 2

(b) 3

(c) 4

(d) 6

15. If the number 5*2 is divisible by 6,then * can be

(a) 2

(b) 3

(c) 6

(d) 7

16. Find the list value of 'x' so that the number 73818x4 is divisible by 8

(a) 2

(b) 3

(c) 4

(d) 1

17. If p713 is divisible by 11, find the value of smallest natural number 'p'

(a) 5

(b) 6

(c) 7

(d) 9

18. which number is divisible by 3 and 4

(a) 465

(b) 471

(c) 156

(d) 628

19. Is 525 divisible by 5?

(a) Yes

(b) No

20. Is 718 divisible by 6?

(a) Yes

(b) No

21. 40,500 is divisible by 6

(a) True

(b) False

22. The divisibility rule for 9 is....

(a) Divide the whole thing by nine.

(b) Add the digits together. If the sum is divisible by 9, the original number is divisible by 9.

(c) Multiply the digits together by 9.

(d) Add the digits together and then add 9.

23.How do you know if a number is divisible by 3?

(a) If sum of the digits is a multiple of 3

(b) If it ends in 3,6,or 9

(c) If it is an even number

(d) If it ends in 0 or 5

24. Which of the following is a even number?

(a) 1

(b) 4

(c) 9

(d) 5

25. 252 is divisible by 6 because:

(a) The digits add up to 6

(b) The sum of the digits is divisible by 6

(c) Its even and the sum of the digits is divisible by 3

(d) The sum of the digits is even

VI. DIVISION ALGORITHM: If we divide a number by another number, the

Dividend=(Divisor × Quotient)+Remainder

VII. SOME SERIES:

(i) $(1 + 2 + 3 + ... + n) = \frac{1}{2}n(n + 1)$

(ii) $(1^2 + 2^2 + 3^2 + ... + n^2) = \frac{1}{6}n(n + 1)(2n + 1)$

(iii) $(1^3 + 2^3 + 3^3 + ... + n^3) = \frac{1}{4}n^2(n + 1)^2$

VIII.BASICS FORMULAE:

(i) $(a + b)^2 = a^2 + b^2 + 2ab$

(ii) $(a - b)^2 = a^2 + b^2 - 2ab$

(iii) $(a + b)^2 - (a - b)^2 = 4ab$

(iv) $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

(v) $a^2 - b^2 = (a + b)(a - b)$

(vi) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

(vii) $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$

(vii) $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$

(vii) $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

(vii) If $(a + b + c) = 0 \Longrightarrow (a^3 + b^3 + c^3) = 3abc$

QUESTIONS:

Q. Find the difference between the local value and the face value of 9 in the number 5798432 .

ANS: (Local value of 9)–(Face value of 9)=90000 – 9 = 89991.

Q. What is the place value of 2 in the numeral 6428753.

Ans: The place value of 2 is 20000.

Q.Which of the following are prime numbers?
(i) 187 (ii) 173 (iii)347 (iv)437 (v)811.

Answer: We know that $14^2 > 187$, here $k = 14$, Prime numbers upto 14 are 2,3,5,7,11,13. Out of these 11 divides 187. therefore 187 is not a prime number.

similarly(iii), $19^2 > 347$, here $k = 19$, prime numbers upto 19 are 2,3,5,7,11,13,17. no one of these divides 347 completely. Therefore 347 is a prime number. Q. Replace the question mark by a suitable number in each of the following:

(i)9873+? = 13200 (ii) 7048-?=3899 (iii) ?-1498=2331

Answer:(i) Let $9873 + x = 13200$
then, $x = (13200 - 9873) = 3327$

(ii) Let 7048-x=3899
then,x=7048-3899=3149

(iii) Let $x - 1498 = 2331$,
then, x=2331+1498=3829